# Controlled Quantum Teleportaion*

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**Abstract**

A theoretical scheme for controlled quantum teleportation is presented, using the entanglement property of **GHZ** state.

## 1 Introduction

In the field of quantum computation and communication, quantum channel plays an important role. In 1993, *Bennet* et.al.[1] presented the theory of quantum teleportation, which provides a theoretical basis for the construction of quantum channels. The first experiment of quantum teleportation was accomplished by *Boumeester* et.al.[2] in 1997, which was generally acknowledged as a milestone in the field of quantum information. In that work, the technique of *Bell*-state analysis[3, 4] was utilized. In 1999, the first three-photon entangle state(**GHZ** state[5]) was experimentally realized[6, 7].

In this paper, the entanglement property of **GHZ** state is utilized to design a theoretical scheme for controlled quantum teleportation. According to the scheme, a third side is included, so that the quantum channel is supervised by this additional side. The signal state cannot be transmitted unless all three sides agree to cooperate.

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## 2 Controlled quantum teleportation

Suppose *Alice*, *Bob* and *Charlie* share a **GHZ** state[5],

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)_{ABC}$$

and *Alice* is to teleport an unknown signal state to *Bob*. The signal was originally carried by qubit $D$,

$$|\psi\rangle_D = \alpha\,|0\rangle_D + \beta\,|1\rangle_D$$

The state for the whole system(four qubits) can be expressed as:

$$\begin{aligned}
|\psi\rangle_{DABC} &= (\alpha\,|0\rangle + \beta\,|1\rangle)_D \otimes \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)_{ABC} \\
&= \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{DA} \otimes (\alpha\,|00\rangle + \beta\,|11\rangle)_{BC} \\
&\quad +\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)_{DA} \otimes (\alpha\,|00\rangle - \beta\,|11\rangle)_{BC} \\
&\quad +\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)_{DA} \otimes (\beta\,|00\rangle + \alpha\,|11\rangle)_{BC} \\
&\quad +\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)_{DA} \otimes (-\beta\,|00\rangle + \alpha\,|11\rangle)_{BC}
\end{aligned}$$

Now *Alice* perform a *Bell*-state measurement[3, 4] on qubits $DA$. After that, she will broadcast the result of her measurement, so that qubits $BC$ can be transformed(According to the four possible results $\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{DA}$, $\frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)_{DA}$, $\frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)_{DA}$ and $\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)_{DA}$, the corresponding transformations are $I_B \otimes I_C$, $I_B \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|)_C$, $(|0\rangle\langle 1| + |1\rangle\langle 0|)_B \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|)_C$ and $(|0\rangle\langle 1| + |1\rangle\langle 0|)_B \otimes (|0\rangle\langle 1| - |1\rangle\langle 0|)_C$, respectively.) by *Bob* and/or *Charlie* to a common form:

$$|\psi\rangle_{BC} = (\alpha\,|00\rangle + \beta\,|11\rangle)_{BC}$$

Considering this state, we can see that, at this moment, neither *Bob* nor *Charlie* can obtain the original signal state $\alpha\,|0\rangle + \beta\,|1\rangle$ without the cooperation of the other one.

If *Charlie* would like to help *Bob* for the teleportation, he should just measure his portion of $BC$, namely qubit $C$, on the bases of $\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_C$ and $\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)_C$, and transfer the result of his measurement to *Bob* via a classical channel. Here the state of qubits $BC$ can be written as:

$$\begin{aligned}
|\psi\rangle_{BC} &= (\alpha\,|00\rangle + \beta\,|11\rangle)_{BC} \\
&= \frac{1}{\sqrt{2}}\cdot(\alpha\,|0\rangle + \beta\,|1\rangle)_B \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)_C \\
&\quad +\frac{1}{\sqrt{2}}\cdot(\alpha\,|0\rangle - \beta\,|1\rangle)_B \otimes \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)_C
\end{aligned}$$

As soon as *Bob* is informed *Charlie*'s result, he can perform an appropriate unitary transformation(According to the two possible results $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_C$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_C$, the corresponding transformations are $I_B$ and $(|0\rangle\langle 0| - |1\rangle\langle 1|)_B$, respectively.) on qubit $B$ to obtain the original signal state,

$$|\psi\rangle_B = \alpha\,|0\rangle_B + \beta\,|1\rangle_B$$

The feature of this scheme is that teleportation between two sides depends on the agreement of the third side. It is therefore named "Controlled Quantum Teleportation".

# 3 Conclusion

The difference of the scheme presented in this paper with the original quantum teleportation scheme[1] is that a third side(*Charlie*) is included, who may participate the process of quantum teleportation as a supervisor. Without the cooperation(permission) of *Charlie*, *Bob* cannot get the signal state from *Alice* by himself.

This property of the scheme can be utilized to construct controlled quantum channels, which may be useful in the future quantum computers.

# References

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